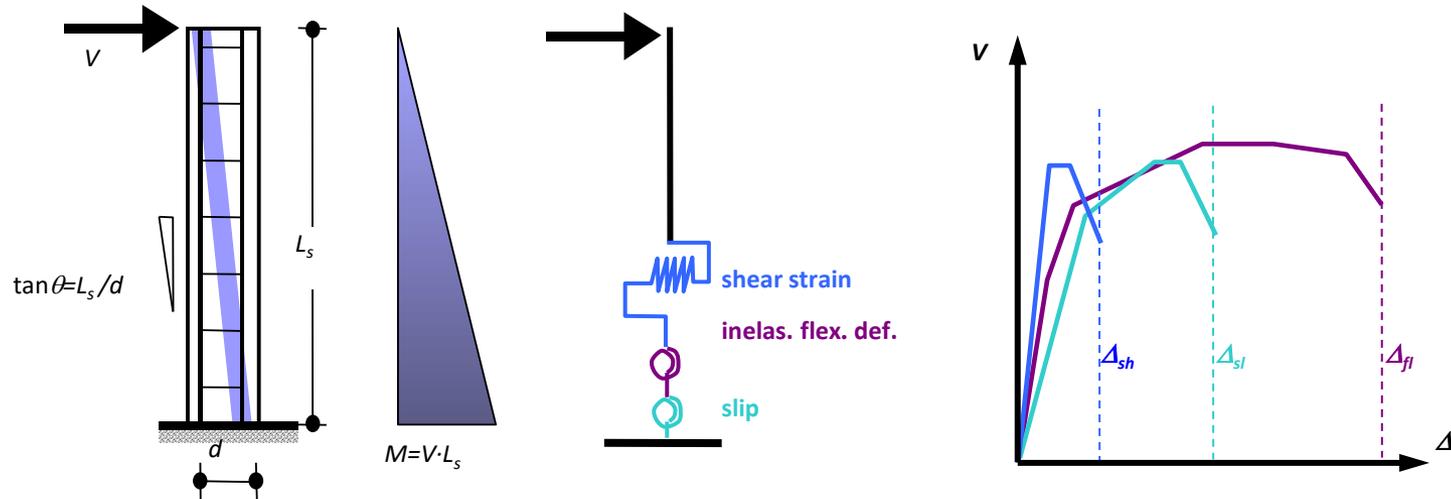


“PHAETHON: Software for Analysis of Shear-Critical Reinforced Concrete Columns”

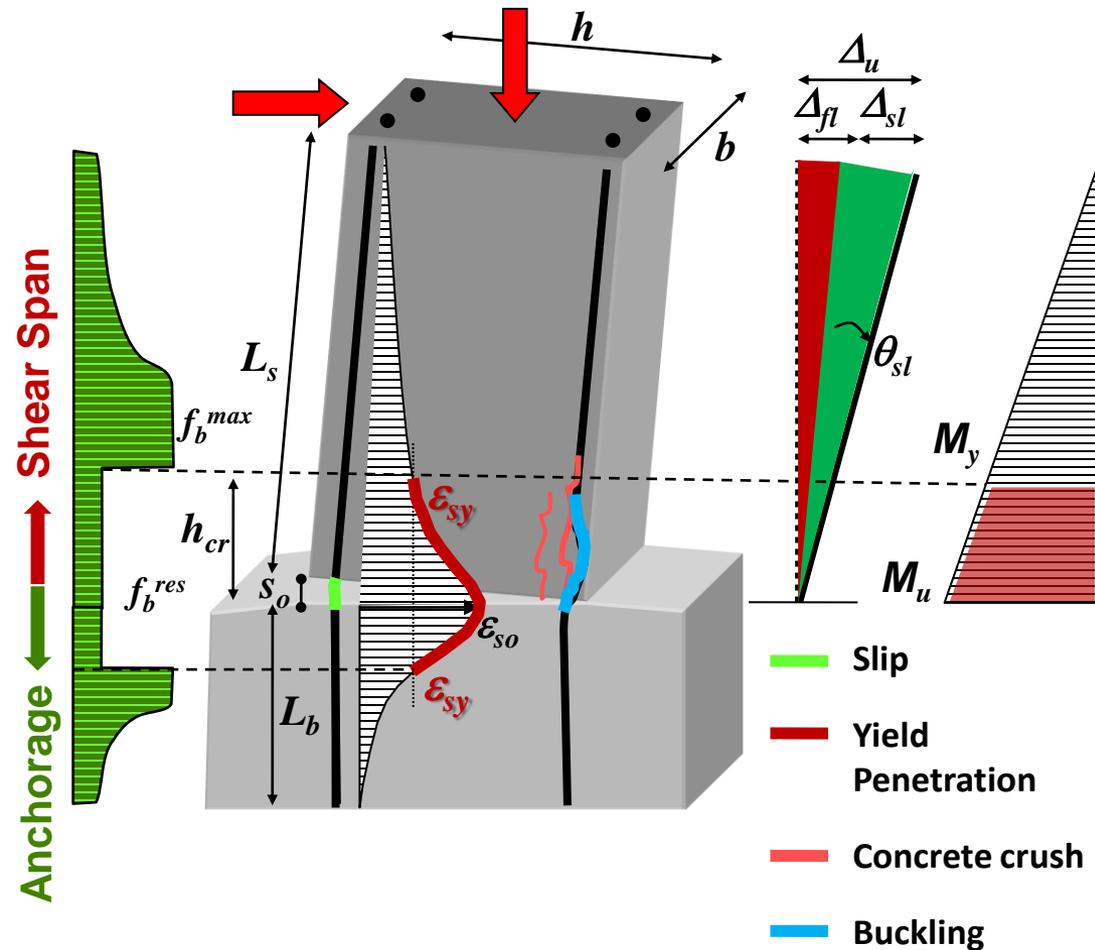
Dr.-Ing. Konstantinos G. Megalooikonomou
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GFZ German Research Centre for Geosciences
Helmholtz Centre Potsdam, Potsdam, Germany
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Research Goal: Seismic Simulation of RC Columns for all possible failure modes



- **The static relationship between shear force and flexural moment** in the span of the cantilever is **identical to that occurring over the length of the actual frame member** extending from the point of contraflexure (zero moment) to the fixed end support.
- **Deformations are owing to flexure, shear action, and pull-out slip** of the reinforcement from the support or lap splice. These mechanisms of behaviour are considered to **act in series**, therefore **their effects are considered additive**, as implied by the mechanical analogue of above Figure, used in computer simulations of inelastic RC Members.

Column Rotation Due to Pull-Out of Tensile Reinforcement:



- The development of yielding flexural moment in plastic hinges of frame elements is synonymous with **yielding strain penetration in shear span and anchorage.**

- Yield penetration destroys interfacial bond between bar and concrete:

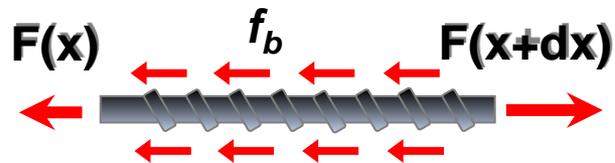
➔ **Reduction of column plastic rotation due to flexure** (reduction of strain development capacity of the reinforcement)

➔ **Increase of bar pull-out contribution in the total column rotation.**

Reinforcement to Concrete Bond ⇔ Tension - Stiffening.

The basic equations that describe force transfer lengthwise from a bar to the surrounding concrete through bond:

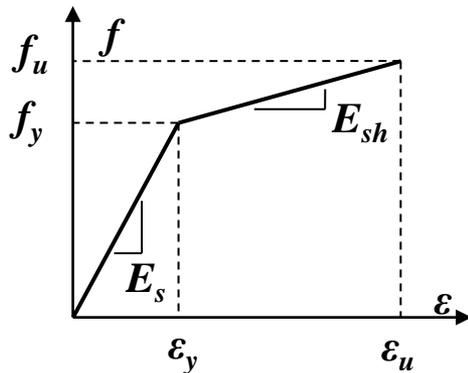
- force equilibrium applied to an elementary bar segment of length dx
- Kinematic relationship: The slip of the bar as the difference of the developed strains by the two materials (Tassios and Yannopoulos 1981, Filippou et. al. 1983):



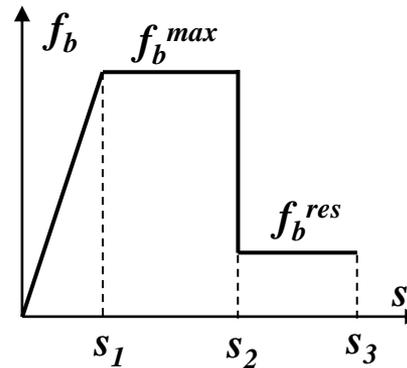
$$\frac{df}{dx} = -\frac{4}{D_b} f_b$$



$$\frac{ds}{dx} = -(\epsilon - \epsilon_c) \cong -\epsilon$$

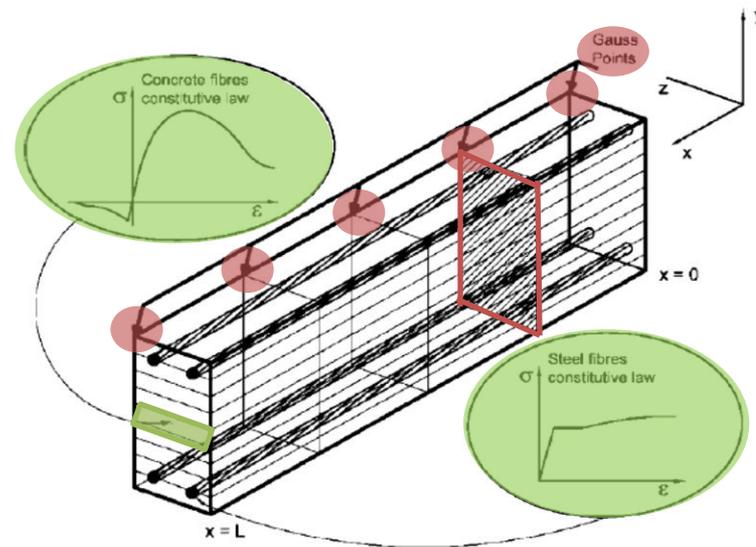


Steel reinforcing bar



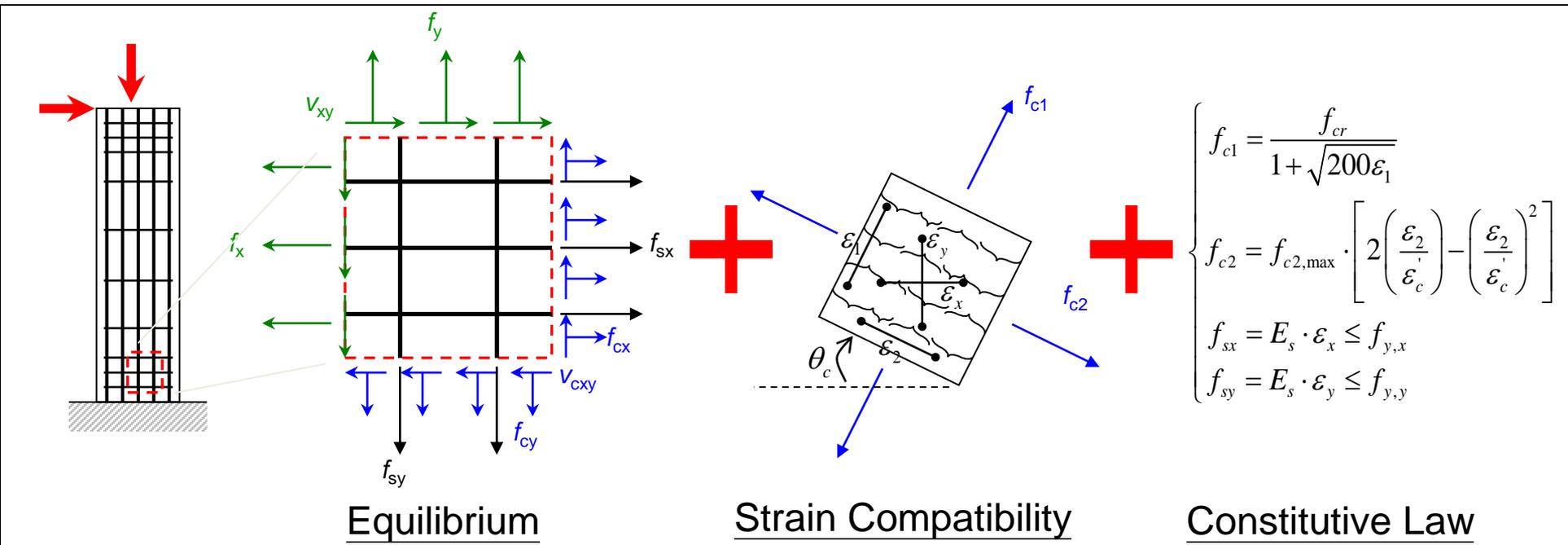
local bond - slip

Nonlinear Analysis with fiber beam-column elements :

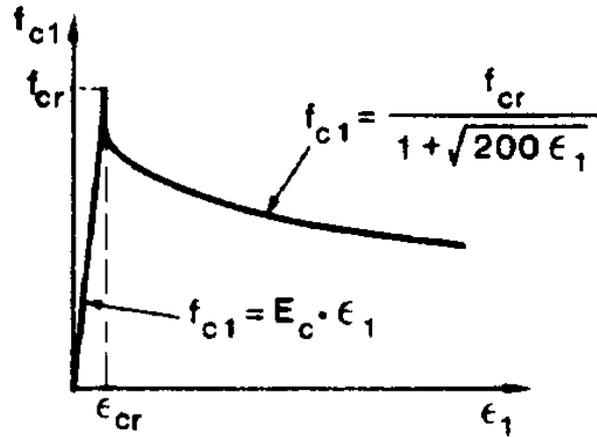


- The beam – column element is discretized in **integration points – sections**.
- It is based in the discretization of the sections of the element in **layers/fibers** where through appropriate **constitutive laws** the forces of the section are determined.
- These are distributed inelasticity models that can be force- or displacement -based.
- In order to evaluate flexural response those elements are based on Euler – Bernouli beam theory but **to evaluate shear- flexure interaction they are based on Timoshenko beam theory**.

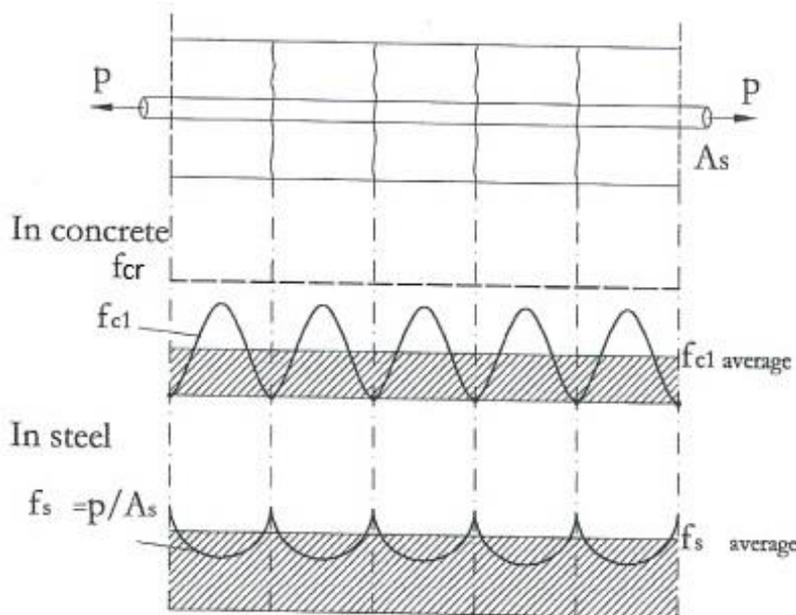
Concrete law – Modified Compression Field Theory (MCFT – Vecchio & Collins 1986):



Bond – Modified Compression Field Theory (MCFT – Vecchio & Collins 1986):

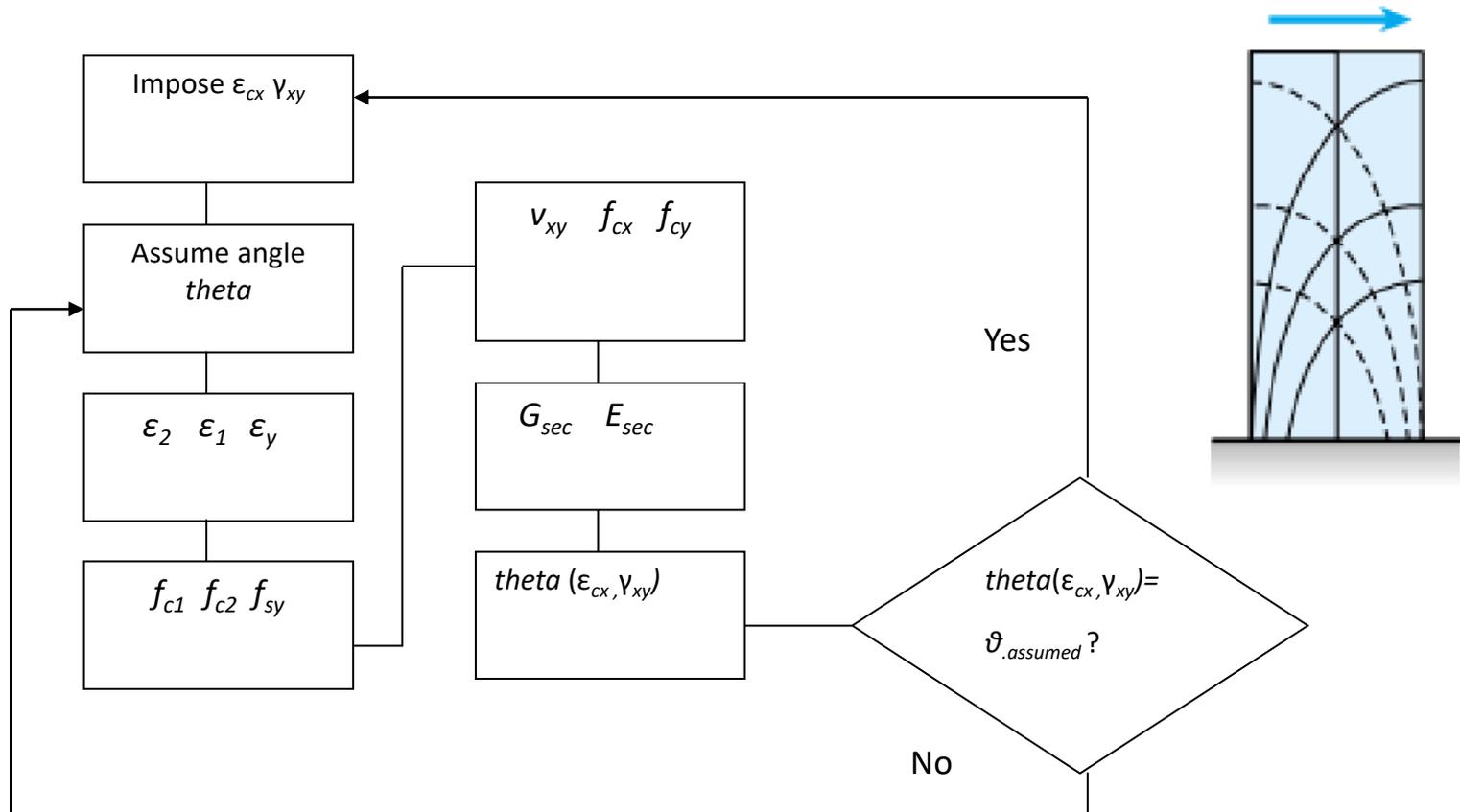


- Due to the influence of bond, tensile stresses can develop in the concrete between cracks. To model this phenomenon, which is referred to in the literature as **“tension stiffening”**, the concrete tensile stress is assumed to decay from the tensile strength as principal strain increases.



- It is assumed that the **average tensile concrete stress**, $f_{c1 \text{ average}}$ is transmitted across cracks. This implies that **stress in the reinforcement** increases in proximity of cracks but it **is limited by yielding value**.

Fiber Stresses – Iterative procedure based on MCFT:



**Root search based on numerical
method Regula Falsi**

Section State Determination:

Section Forces

$$f_s(x) = \int B_s^T(y) \cdot \sigma(x, y) dA$$

$$f_s(x) = \begin{Bmatrix} N \\ M \\ V \end{Bmatrix} \quad \sigma(x, y) = \begin{Bmatrix} \sigma_x \\ \tau_{xy} \end{Bmatrix}$$

$$B_s(y) = \begin{bmatrix} 1 & -y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N = \int \sigma_x dA = \text{Axial Force}$$

$$V = \int \tau_{xy} dA = \text{Shear Force}$$

$$M = - \int y \sigma_x dA = \text{Moment}$$

Section Stiffness

$$k_s = \begin{bmatrix} \frac{\partial f_{s1}}{\partial e_1} & \frac{\partial f_{s1}}{\partial e_2} & \frac{\partial f_{s1}}{\partial e_3} \\ \frac{\partial f_{s2}}{\partial e_1} & \frac{\partial f_{s2}}{\partial e_2} & \frac{\partial f_{s2}}{\partial e_3} \\ \frac{\partial f_{s3}}{\partial e_1} & \frac{\partial f_{s3}}{\partial e_2} & \frac{\partial f_{s3}}{\partial e_3} \end{bmatrix}$$

$$\sigma(x, y) = \begin{Bmatrix} \sigma_x \\ \tau_{xy} \end{Bmatrix}$$

$$\varepsilon(x, y) = \begin{Bmatrix} \varepsilon_x \\ \gamma_{xy} \end{Bmatrix}$$

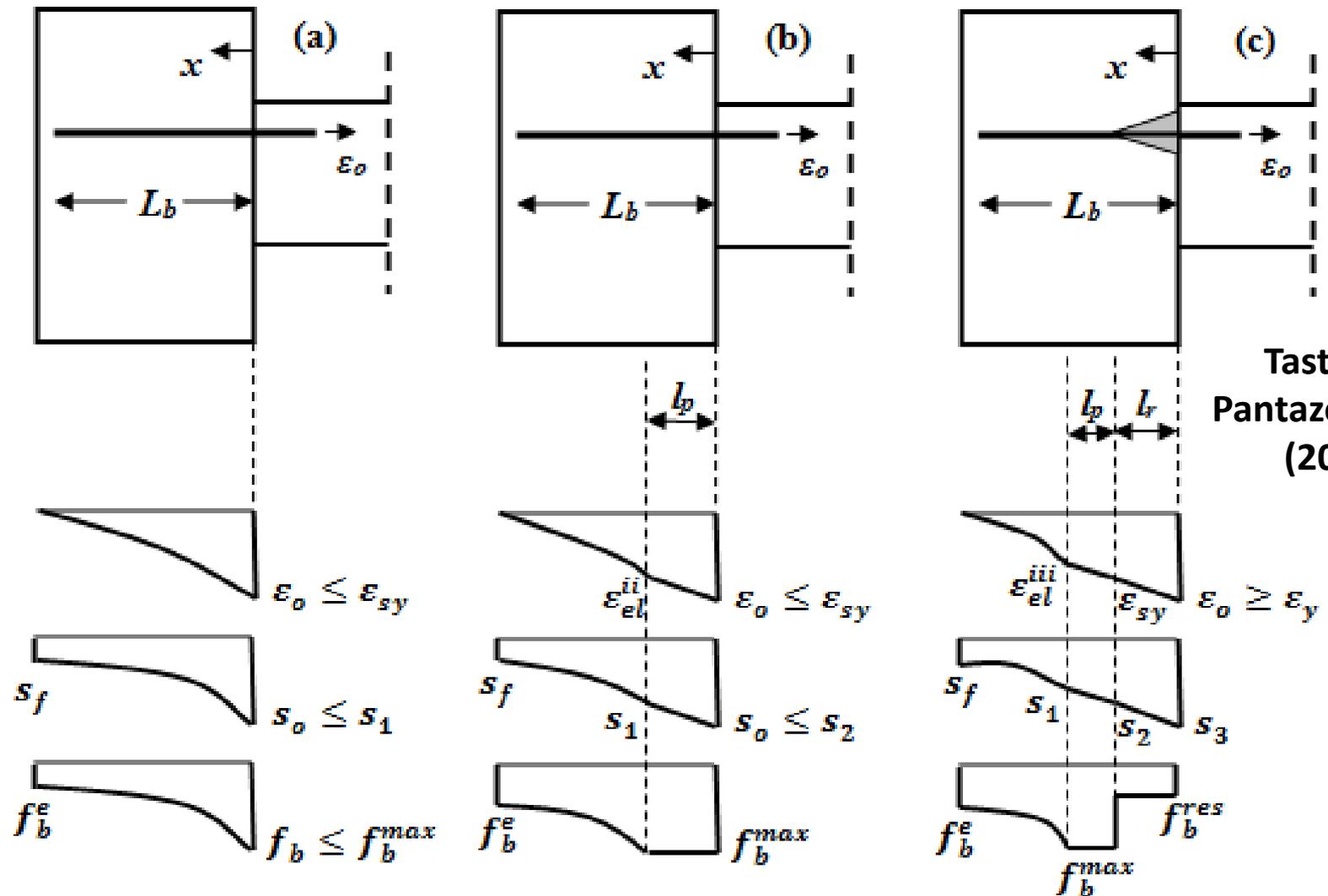
$$\frac{d\sigma(x, y)}{d\varepsilon(x, y)} = \begin{bmatrix} E_m & 0 \\ 0 & G_m \end{bmatrix}$$

$$k_s = \frac{\partial f_s}{\partial e} = \int B_s^T(y) \cdot \frac{d\sigma(x, y)}{d\varepsilon(x, y)} \cdot \frac{\partial \varepsilon(x, y)}{\partial e} dA = \int B_s^T(y) \cdot \frac{d\sigma(x, y)}{d\varepsilon(x, y)} B_s(y) dA$$

$$N = \sum_{i=1}^{n.layer} \sigma_x^i A^i \text{ and } V = \sum_{i=1}^{n.layer} \tau_{xy}^i A_s^i \text{ and } M = - \sum_{i=1}^{n.layer} \sigma_x^i y^i A^i$$

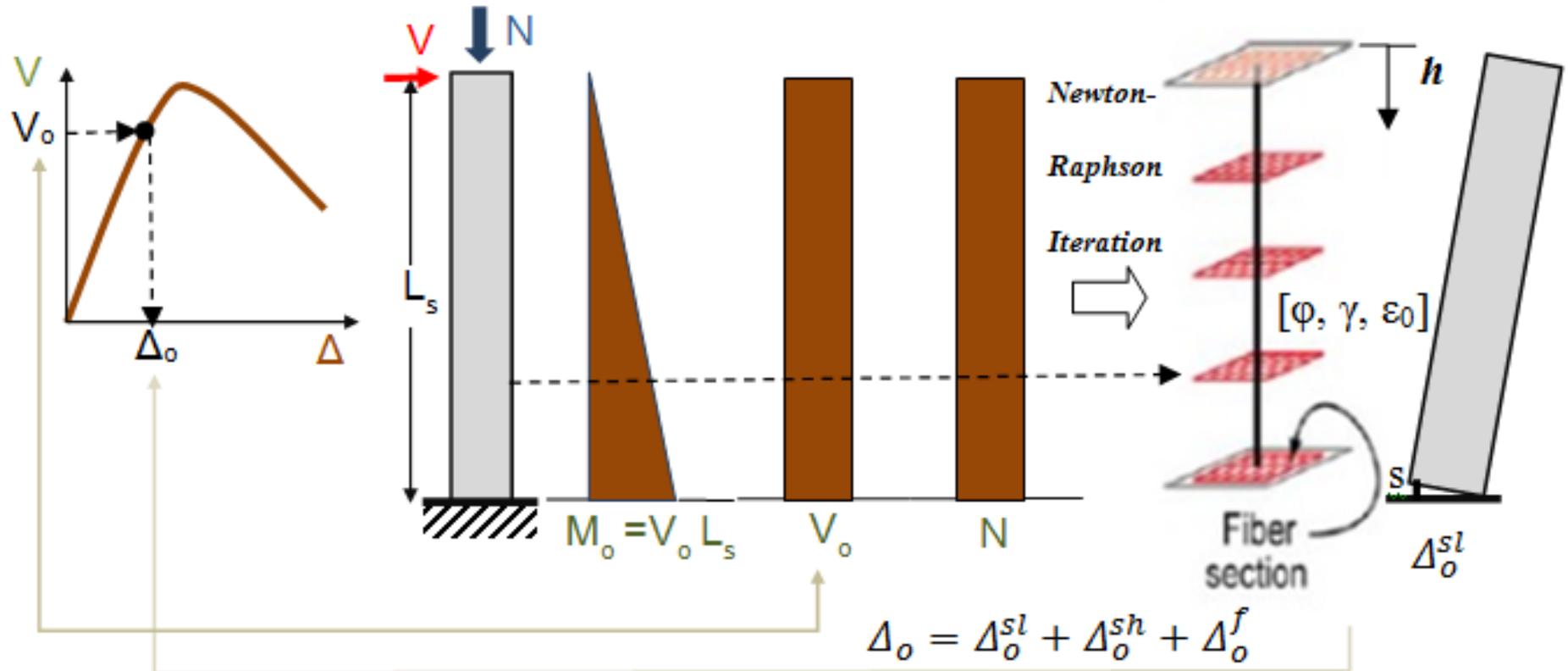
Note: Parabolic Shear Strain Distribution along Section height with maximum value (γ_{xy}) at the neutral axis.

Strain, Slip and Bond Distributions in the Anchorage Length:



Tastani &
Pantazopoulou
(2013)

Capacity Curve of Shear-Critical RC Column - Phaethon:



$$\Delta_o^{sh} = \int_0^{L_s} \gamma(h) \cdot dh$$

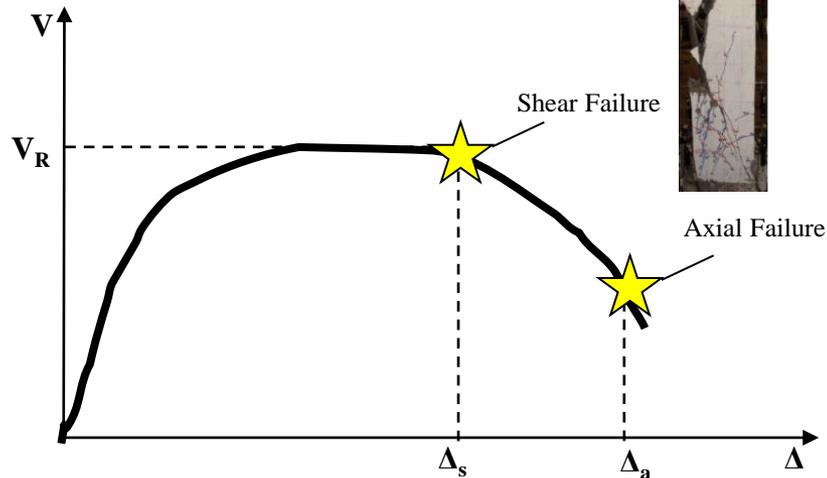
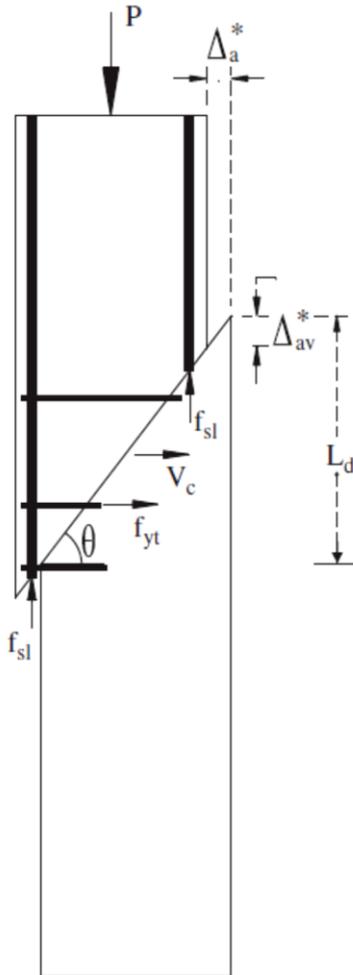
$$\Delta_o^f = \int_0^{L_s} \varphi(h) \cdot h \cdot dh$$

$$\Delta_o^{sl} = \frac{s}{(d - 0.4c_x)} \cdot L_s$$

Drift at Axial Failure (Elwood and Moehle, 2005):

- Elwood and Moehle (2005)

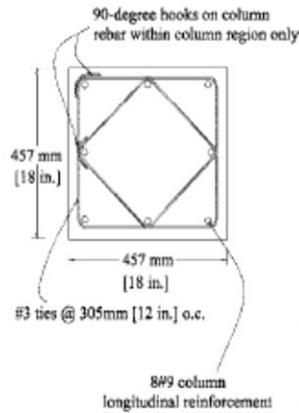
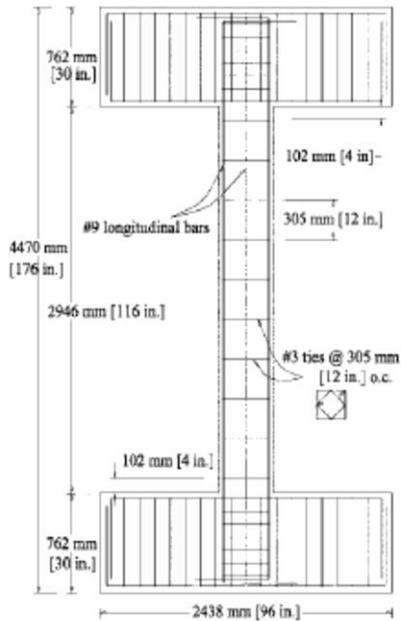
$$\left(\frac{\Delta}{L}\right)_{axial} = \frac{4}{100} \frac{1 + (\tan 65^\circ)^2}{\left[\tan 65^\circ + P \cdot \left(\frac{s}{A_{st} \cdot f_{yt} \cdot d_c \cdot \tan 65^\circ}\right)\right]}$$



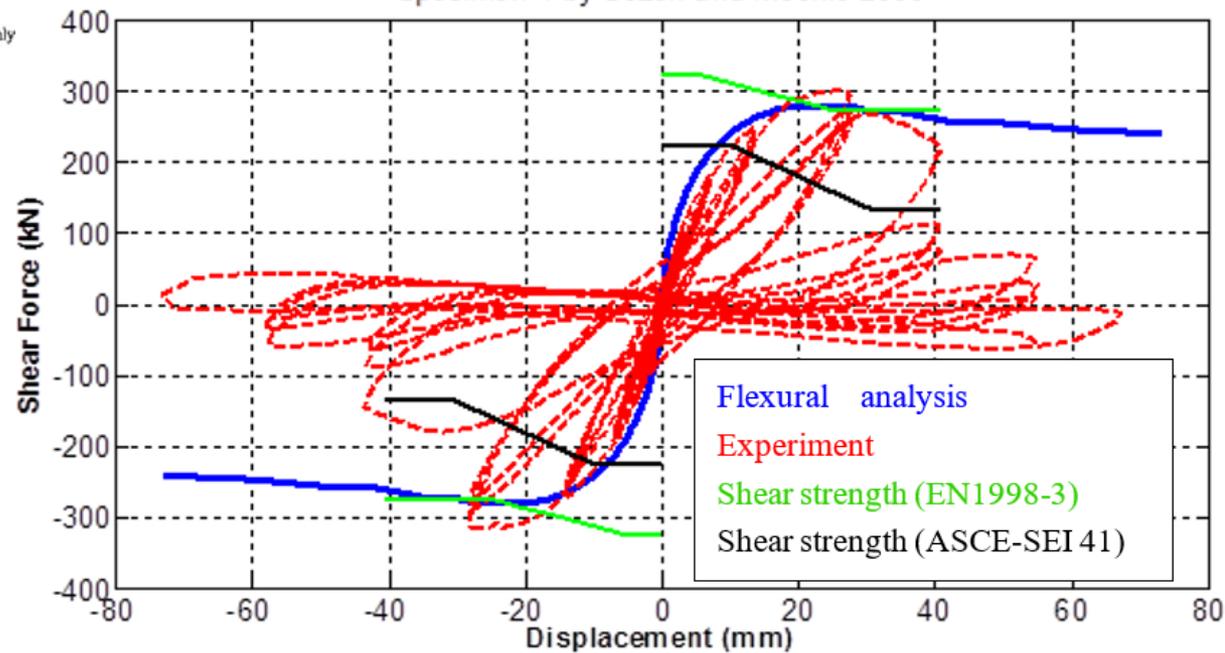
Correlation with Experimental Results - Specimen 1 (Sezen & Moehle, 2006):

Cantilever Column

Specimen 1 by Sezen and Moehle 2006



Sezen & Moehle (2006)



Correlation with Experimental Results – Phaethon - Specimen 1:

Phaethon

File About

Geometry

Cross-Section Type
 Rectangular Circular

h (mm) = 457.2 D (mm) =

b (mm) = 457.2

Concrete

fc (MPa) = 21.1 fcr (MPa) = 1.516

epc0 = 0.002 ecr = 0.00007184

Reinforcement Longitudinal

n.Mid = 1 nBars = 4

Area of section reinforcement layers

Dbar (mm) =

Es (MPa) = 200000 fy (MPa) = 434 Eh (MPa) = 0.0 Cov (mm) = 65.13

Transversal

rsy = 0.0025 fyy (MPa) = 476 Sp (mm) = 304.8 n.StLegs = 3.41 Dt (mm) = 9.525

Solution Procedures

nIP.Sec = 10 IntTyp.Sec = Midpoint N (kN) = 667 Compressive N Tensile N

Pushover Moment - Curvature

Ls (mm) = 1473 nIP.Ls = 5

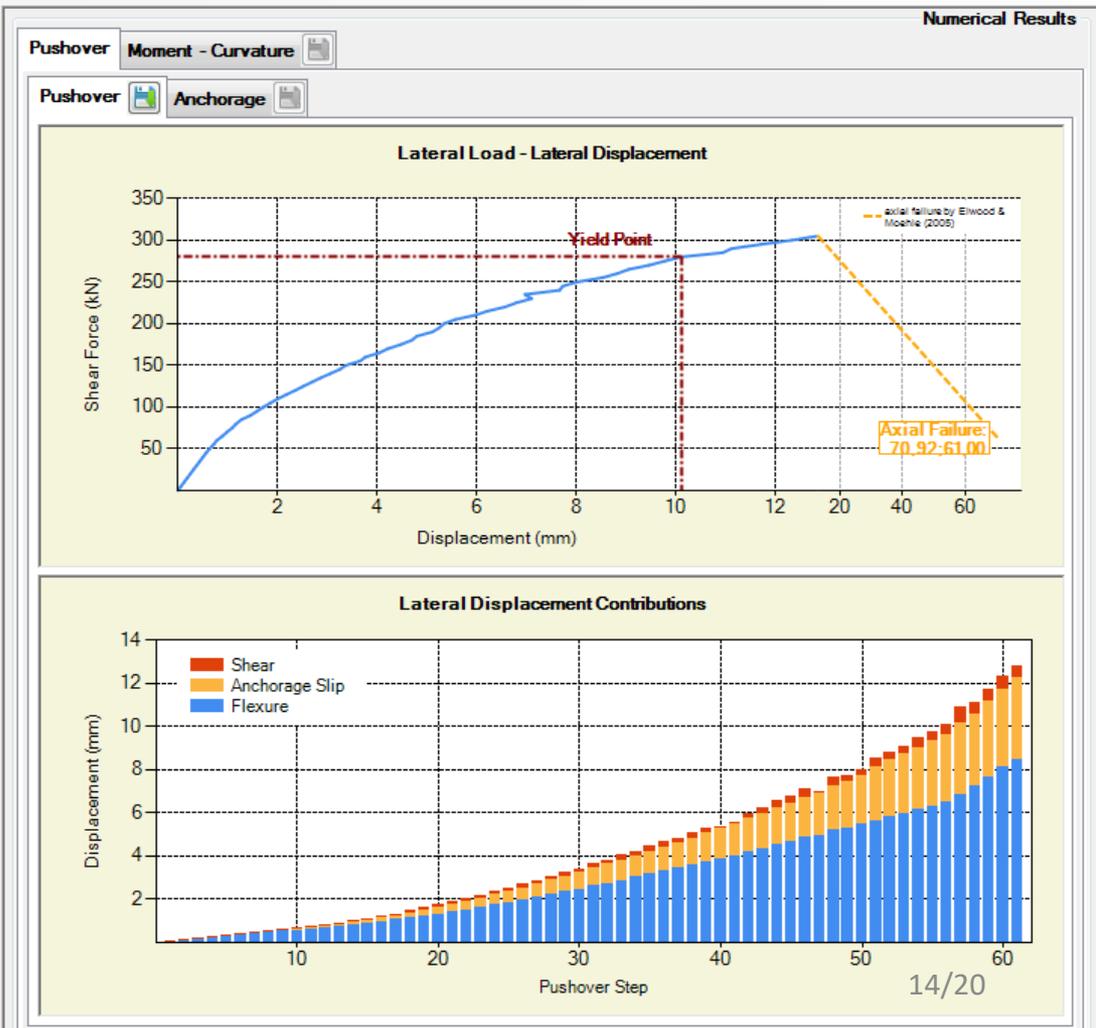
NoPushSteps = 65 IntTyp.Ls = Gauss-Lobatto

PushStep (kN) = 5

Anchorage

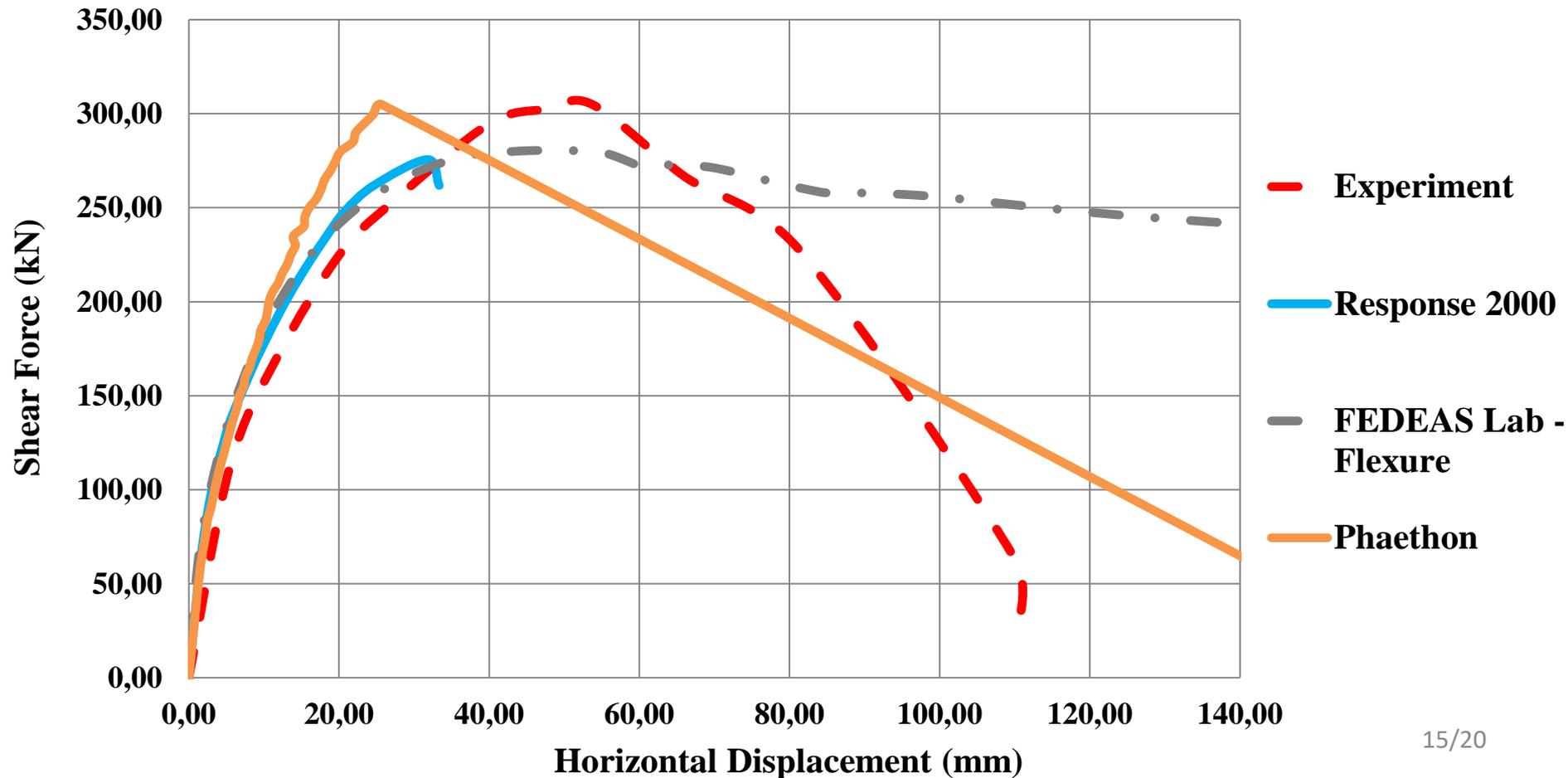
fc (MPa) = 21.1 Db (mm) = 28.7 Lb (mm) = 1121

Solve

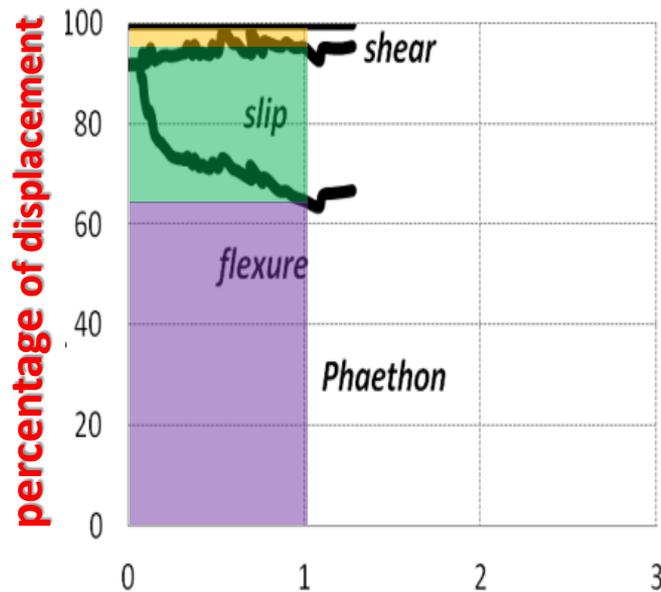


Correlation with Experimental Results – Phaethon - Specimen 1:

Specimen 1 (Double Curvature) tested by Sezen & Moehle (2006)

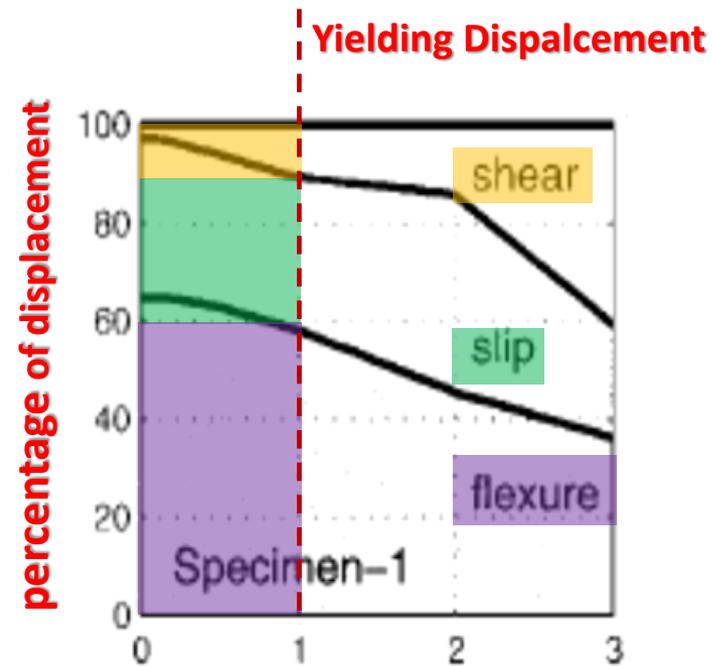


Correlation with Experimental Results- Phaethon - Specimen 1:



$\Delta_{total} / \Delta_y$

Numerical
Results



$\Delta_{total} / \Delta_y$

Experimental
Results

Phaethon – Anchorage – Specimen 1 (Sezen & Moehle, 2006):

Phaethon

File About

Geometry

Cross-Section Type

Rectangular Circular

h (mm) = 457.2 D (mm) =

b (mm) = 457.2

Reinforcement

Longitudinal

n.Mid = 1

nBars = 4

Area of section reinforcement layers

Dbar (mm)

Concrete

fc (MPa) = 21.1 fcr (MPa) = 1.516

epc0 = 0.002 ecr = 0.00007184

Transversal

rsy = 0.0025

fy (MPa) = 434 fyy (MPa) = 476

Eh (MPa) = 0.0 Sp (mm) = 304.8

Cov (mm) = 65.13 n.StLegs = 3.41

Dt (mm) = 9.525

Solution Procedures

nIP.Sec = 10 IntTyp.Sec = Midpoint N (kN) = 667

Compressive N
 Tensile N

Pushover **Moment - Curvature**

Ls (mm) = 1473 nIP.Ls = 5

NoPushSteps = 65 IntTyp.Ls = Gauss-Lobatto

PushStep (kN) = 5

Anchorage

fc (MPa) = 21.1

Db (mm) = 28.7

Lb (mm) = 1121

Solve

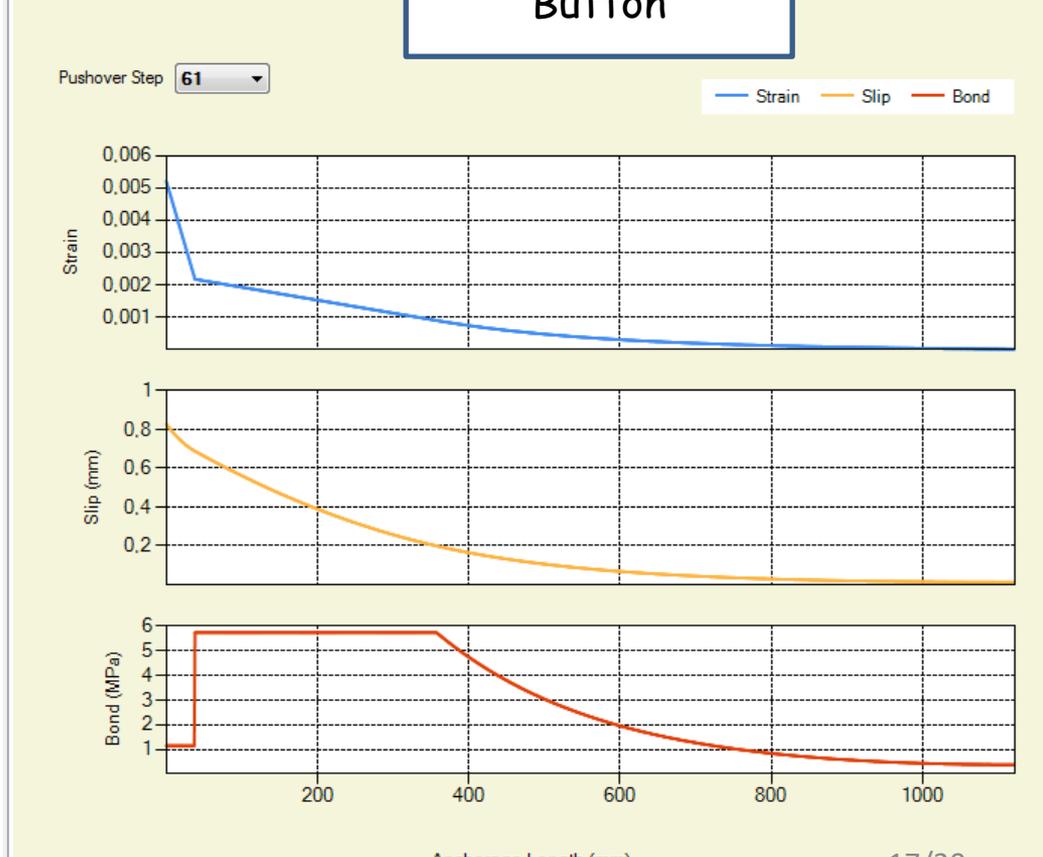
Numerical Results

Pushover **Moment - Curvature** Pushover **Anchorage**

Download Results Button

Pushover Step 61

— Strain — Slip — Bond



Anchorage Length (mm)

Sectional Analysis: Moment – Curvature & Shear Force – Shear Strain (Specimen 1)

Phaethon

File About

Geometry

Cross-Section Type
 Rectangular Circular

h (mm) = 457.2 D (mm) =

b (mm) = 457.2

Concrete

fc (MPa) = 21.1 fcr (MPa) = 1.516

epc0 = 0.002 ecr = 0.00007184

Reinforcement

Longitudinal

n.Mid = 1 n.Bars = 4

Es (MPa) = 200000

fy (MPa) = 434

Eh (MPa) = 0.0

Cov (mm) = 65.13

Area of section reinforcement layers

Dbar (mm) =

Transversal

rsy = 0.0025

fyy (MPa) = 476

Sp (mm) = 304.8

n.StLegs = 3.41

Dt (mm) = 9.525

Solution Procedures

nIP.Sec = 10 Int.Typ.Sec = Midpoint N (kN) = 667

Compressive N
 Tensile N

Pushover Moment - Curvature

phi Incr. (1/mm) = 0.000001

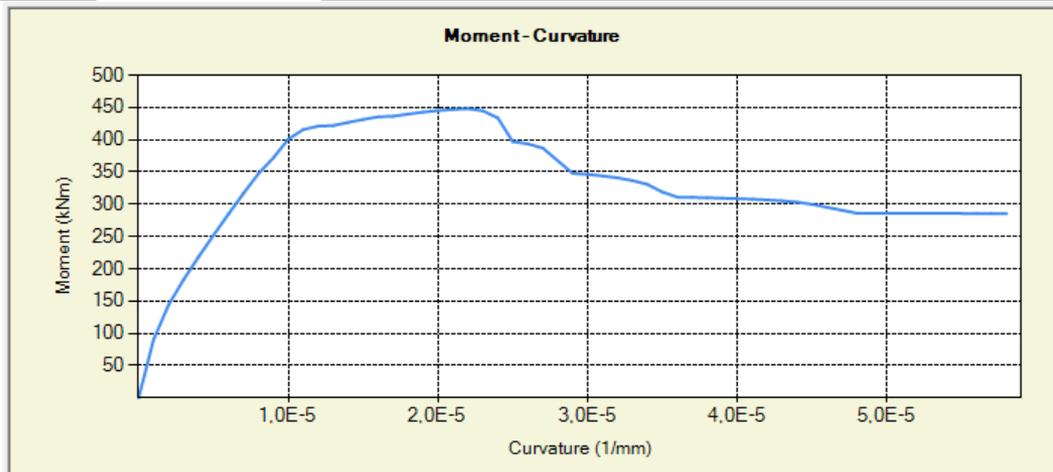
gamma Incr. = 0.00001

NoSteps = 65

Solve

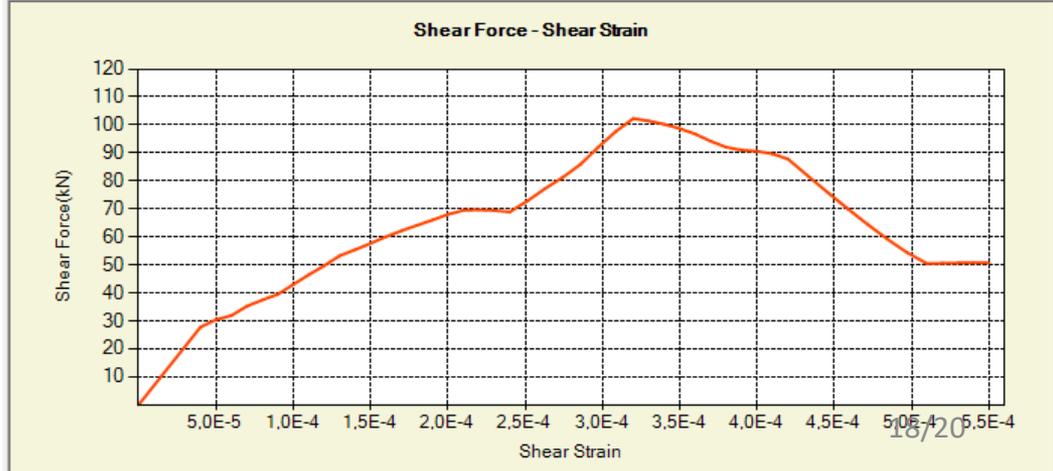
Numerical Results

Pushover Moment - Curvature



Curvature (1/mm)	Moment (kNm)
0.0E+00	0
1.0E-5	350
2.0E-5	450
3.0E-5	350
4.0E-5	300
5.0E-5	300

Shear Force - Shear Strain

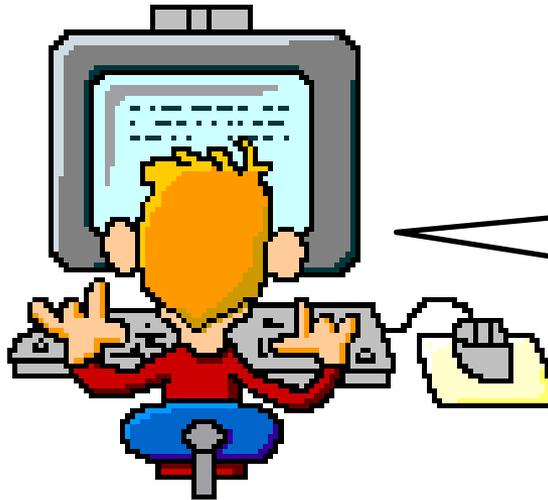


Shear Strain	Shear Force (kN)
0.0E+00	0
5.0E-5	30
1.0E-4	45
1.5E-4	60
2.0E-4	70
2.5E-4	70
3.0E-4	85
3.2E-4	100
3.5E-4	95
4.0E-4	90
4.5E-4	75
5.0E-4	55
5.5E-4	50

Conclusions:

- A **Windows-based software** was developed for fiber-based, distributed nonlinearity analysis of prismatic frame elements undergoing lateral sway such as would occur during an earthquake.
- The formulation was extended to fiber-type analysis with distributed nonlinearity also considering the **exact Timoshenko beam theory** whereby shear deformations are explicitly considered in the state determination.
- **Moment, shear and axial load interaction** were considered in calculating the resistance curve for a number of different column cases that underwent flexure shear or purely shear dominated mode of failure, and the **distinct contributions of the many contributing sources of column deformation (curvature, shear angle, axial elongation, pullout rotation)** were illustrated through the developed algorithm.
- **Good correlation with experimental results from the literature.**

THANK YOU FOR YOUR ATTENTION!



**ANY
QUESTIONS
?**

ACKNOWLEDGEMENTS

This work has been carried out with the financial support of the Alexander S. Onassis Public Benefit Foundation.