COMPDYN 2015
5th ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, V. Papadopoulos, V. Plevris (eds.)
Crete Island, Greece, 25–27 May 2015

NUMERICAL MODELING OF FRP - RETROFITTED CIRCULAR RC COLUMNS INCLUDING SHEAR

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Keywords: FRP, RC column, circular section, confinement, shear deformation.

Abstract. *The Fiber Reinforced Polymer (FRP)-confined concrete model contained in a well-known Bulletin by the International Federation for Structural Concrete (fib) has been enhanced to take into account the superposition of the confining effects of the already existing steel reinforcement with that of the FRP jacketing applied when retrofitting reinforced concrete (RC) columns. Columns are here modeled with a fiber-based nonlinear beam- column element (with displacement formulation) in which the constitutive law for concrete presented in this paper is implemented. This allows for the immediate incorporation of shear strains (uncoupled from the normal ones) at the material level. The averaged response of the two different regions -concrete core and concrete cover-in the cross-section allows the assignment of a unique stress-strain law for all the fibers/layers of the circular section. Correlation with an experimental study is performed to validate the proposed iterative procedure.*

1 INTRODUCTION

Several strong earthquakes attacked California U.S.A., Kobe Japan, and Ji-Ji Taiwan. These major earthquakes have caused severe structural damage, and even collapse which have resulted in the necessity of seismic retrofit of existing structures. Confining wraps or jackets to rehabilitate and strengthen existing concrete columns has proven to be an efficient technique for seismic retrofit of structures. Fiber Reinforced Polymers (FRPs) to retrofit RC columns have been found to be beneficial for the compressive strength, shear strength, flexural strength and displacement ductility of RC columns.

It is well known that columns play a key role in determining the overall structural performance. Therefore, it is essential to develop a suitable analytical tool to estimate their structural behavior, able to detect all failure mechanisms such as shear, and bending and possibly to include the effect of FRP confinement as a retrofitting measure.

The behavior of reinforced concrete columns in shear and bending has been studied for decades. In case of flexural behavior, traditional section analysis, or the more precise method, fiber model in one-dimensional stress field gives acceptable predictions in terms of ultimate strength and yielding deformation. Performance of reinforced concrete columns dominated in shear or shear-flexure cannot be estimated by applying only section analysis because shear behavior is not taken into account. For evaluating the shear response of structural elements, such as beams and columns, many analytical models and theories have been conceived in the past [1].

Most of the state-of-the-art on seismic design and assessment procedures proposed recently for common engineering practice requires some kind of nonlinear analysis either static or dynamic. These nonlinear analyses are mostly carried out using frame elements with different levels of accuracy. Two main approaches are traditionally used and classified as lumped-plasticity and distributed-inelasticity models. For the latter, the well known fiber beam elements provide results that are particularly accurate when studying the cyclic behavior of RC structures.

Several fiber beam-column elements have been developed with high capability of reproducing axial force and bending coupling. On the other hand, the coupling between the effects of normal force and shear force is not straightforward and hence only few modeling strategies have successfully dealt with it up to now [1].

The most basic theory capable of analyzing beam-column elements is the Euler-Bernoulli approach. The fundamental assumptions are that cross-sections remain plain and normal to the deformed longitudinal axis. The Euler-Bernoulli theory is capable of reproducing correctly the actual response of a beam under combined axial forces and bending moments, while shear forces are obtained from static equilibrium (the effects of the latter on beam deformations are neglected though). When the effects of tangential stresses are important in the response of the element, more refined Timoshenko-like beam theories are more appropriate. Here cross-sections remain plane but not necessarily normal to the deformed longitudinal axis.

The Timoshenko beam theory allows considering the shear strain effect through a constant shear strain distribution along the cross-section, implying that an effective shear area factor is needed in order to be energetically consistent. This factor depends on the section shape and is usually derived for isotropic materials. This implies that being state-dependent, it is far more complex to use in nonlinear analysis of cracked RC sections.

2 ANALYTICAL INVESTIGATION

The mechanical properties of concrete (strength, pseudo-ductility, energy dissipation) are substantially enhanced under a triaxial stress state. In practice, this is obtained by using closed stirrups or spiral reinforcement or even FRP wraps, so that, together with the longitudinal reinforcement, the lateral expansion of concrete is limited. This kind of (passive) confinement improves the material behavior after the initiation of internal cracking, which gives rise to the initiation of expansion.

For low strain values, the stress state in the transverse steel reinforcement is very small and the concrete is basically unconfined. In this range, steel and FRP jacketing behave similarly. That is, the inward pressure as a reaction to the expansion of concrete increases continuously. Therefore, speaking in terms of variable confining pressures corresponding to the axial strain level in the section and active triaxial models defining axial stress-strain curves for concrete subject to constant lateral pressure, it can be stated following the original approach of [2] that the stress-strain curve describing the stress state of the section has to cross all active confinement curves up to the curve with lateral pressure equal to the one applied by the stirrups at yielding. After yielding of stirrups, the lateral pressure is still increasing only due to the FRP jacketing, while the steel lateral pressure remains constant. The corresponding stress-strain curve of the section throughout this procedure converges to a confined-concrete axial stress-strain curve that is associated with a lateral pressure magnitude equal to the tensile strength of the FRP jacket plus the yielding strength of ties (excluding the strain hardening behavior of steel, since ultimate strains of steel are usually much higher than those of FRP jackets). In order to model this behavior, a well-known FRP-confined concrete model [2] has been enhanced to include the steel ties contribution and thus model in a more consistent way circular columns with transverse steel reinforcement and retrofitted with FRP jacketing. The above model was based on an iteration procedure that needed to be modified as in Figure 1.

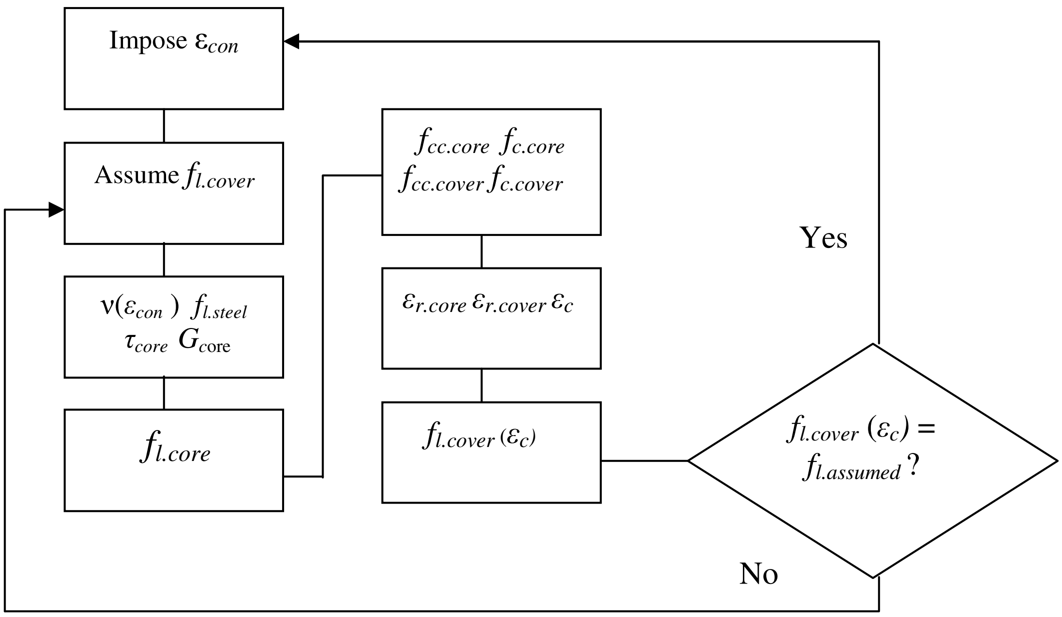

Figure 1: Iterative Procedure.

In the procedure depicted in (Fig.1), after imposing an axial strain on the section, a pressure coming from the FRP jacket is assumed. Then, the Poisson's coefficient until yielding of steel stirrups and the pressure coming from the steel ties is calculated based on the BGL model [3]. Since, this lateral pressure according to BGL model is the solution of the plain stress tensor by the Airy's stress function, the shear stress in the concrete core is also determined along with the shear modulus .Here, also the longitudinal bars' contribution and the arching action between two adjacent stirrups along the column are taken into account (Table 1). Thus, the confining pressure in the concrete core is the summation of the lateral pressures contributed by the two confining systems (FRP and Steel). The fib's model proposal [2] beyond this point is basically used to define the remainder of the parameters declared above, applying that model for the two different regions already mentioned. The focal point of the procedure is in the last step where the confining pressure of the jacket is defined based on the circumferential strain according to Table 1. Finally, cases with partial wrapping have been included too [4] (Table 1). Such approach also permits in cases of repair and retrofit to consider two different concrete strengths, one for the new layer of concrete applied externally and the other for the old existing concrete core which may also be cracked due to former seismic loading [5]. At the end of the procedure, a two-condition failure criterion [5] is incorporated either due to excessive dilation of concrete or due to buckling of longitudinal bars.

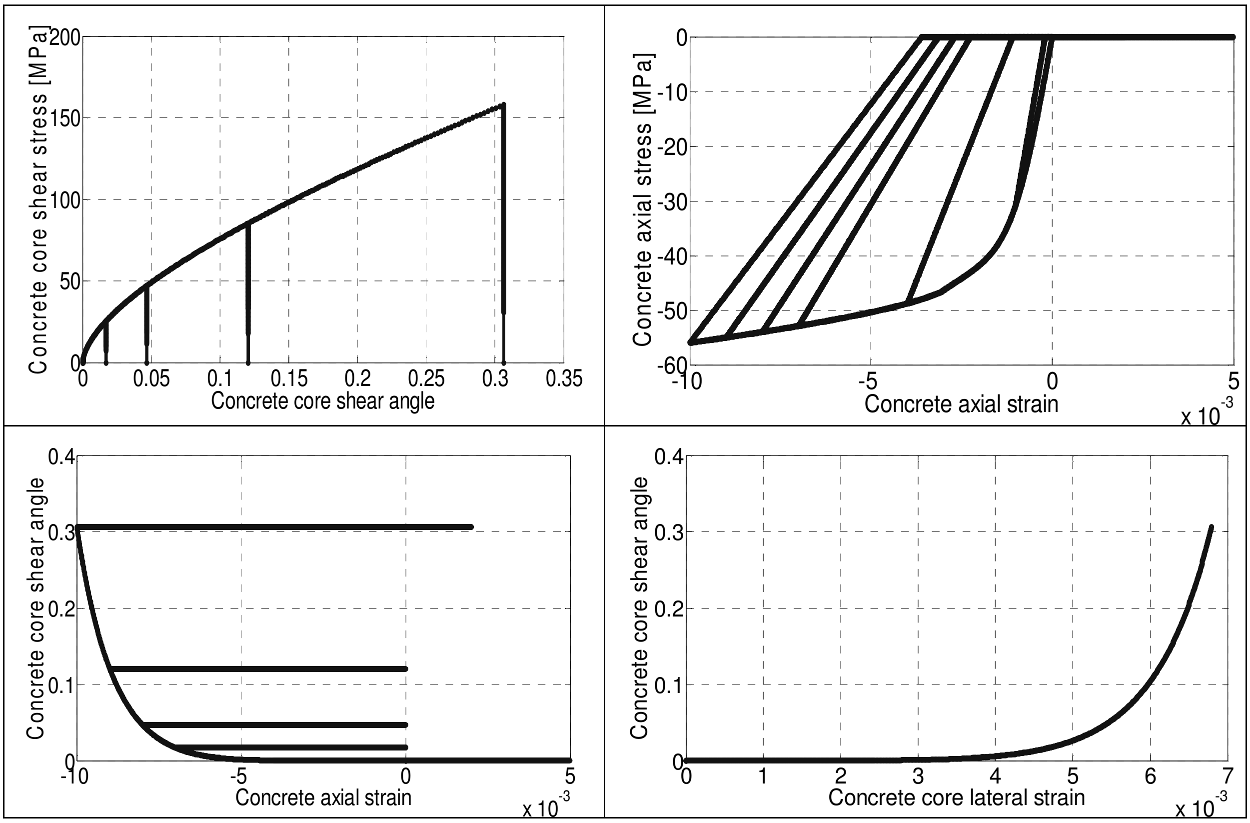

Figure 2: Prediction of concrete behavior with properties similar to ST2NT by Sheikh and Yau [6]

(Braga et al. 2006) [3]	$f_{l.steel}(\varepsilon_{con}) = k_{sl}\dfrac{E_{con}E_s A_{sh}\nu(\varepsilon_{con})}{R_{core}E_{con}S + E_s A_{sh}[1-\nu(\varepsilon_{con})]\cdot[\nu(\varepsilon_{con})\cdot\varepsilon_{con}+1]}\cdot\varepsilon_{con}, \varepsilon_{sh} \prec \varepsilon_{yh}$ $f_{l.steel}(\varepsilon_{con}) = k_{sl}\,0.5\rho_{sh} f_{yh}, \varepsilon_{shu} \geq \varepsilon_{sh} \geq \varepsilon_{yh}$ $\rho_{sh} = \dfrac{4A_{sh}}{D_{core}S},\ \varepsilon_{yh} = \dfrac{f_{yh}}{E_s},\ \varepsilon_{sh} = \varepsilon_{c.core} = \varepsilon_{r.core} = \nu(\varepsilon_{con})\cdot\varepsilon_{con}$ $\nu(\varepsilon_{con}) = \nu_0\left[1+0.2\left(\dfrac{\varepsilon_{con}}{\varepsilon_{c0}}\right)-\left(\dfrac{\varepsilon_{con}}{\varepsilon_{c0}}\right)^2+1.55\left(\dfrac{\varepsilon_{con}}{\varepsilon_{c0}}\right)^3\right],\ \nu_0 = 0.2,\ \varepsilon_{co} = 0.002$ $k_{sl} = \dfrac{45\xi_l^{\,3}}{45\xi_l^{\,3}+\beta\xi_{st}}, \xi_l = \dfrac{D_b}{S}, \beta = \dfrac{D_h}{D_b}, \xi_{st} = \dfrac{2D_h}{\pi R_{core}}$ $\tau_{core} = \dfrac{E_{con}E_s A_{sh}\nu(\varepsilon_{con})}{R_{core}E_{con}S + E_s A_{sh}[1-\nu(\varepsilon_{con})]\cdot[\nu(\varepsilon_{con})\cdot\varepsilon_{con}+1]}\cdot\varepsilon_{con}$ $G_{core} = \dfrac{E_{con}}{2\cdot[1+\nu(\varepsilon_{con})]}$
(Spoelstra and Monti 1999) [2]	$f_c = \dfrac{f_{cc}\cdot x\cdot r}{r-1+x^r},\ x = \dfrac{\varepsilon_{con}}{\varepsilon_{cc}},\ \varepsilon_{cc} = \varepsilon_{co}\left[1+5\left(\dfrac{f_{cc}}{f_{co}}-1\right)\right],\ r = \dfrac{E_{con}}{E_{con}-E_{\sec}}$ $E_{\sec} = \dfrac{f_{cc}}{\varepsilon_{cc}},\ \dfrac{f_{cc}(f_l)}{f_{co}} = 2.254\sqrt{1+7.94\dfrac{f_l}{f_{co}}}-2\dfrac{f_l}{f_{co}}-1.254,$ $\varepsilon_r(\varepsilon_{con}, f_l) = \dfrac{E_{con}\varepsilon_{con} - f_c(\varepsilon_{con}, f_l)}{2\beta f_c(\varepsilon_{con}, f_l)},\ \beta = \dfrac{5700}{\sqrt{\lvert f_{co}\rvert}}-500,\ E_{con} = 5700\cdot\sqrt{\lvert f_{co}\rvert}\,(MPa)$
(14th fib Bulletin) [4]	$k_j = \dfrac{\left(1-\dfrac{S_j}{2D}\right)^2}{1-\dfrac{A_b}{A_g}} \approx \left(1-\dfrac{S_j}{2D}\right)^2$ *(Coefficient for Partial Wrapping)* $f_{l.cover} = \dfrac{1}{2}k_j\rho_j E_j\varepsilon_c, \rho_j = \dfrac{4t_j}{D}$
Iterative Procedure	$f_{l.core} = f_{l.cover} + f_{l.steel},\ f_{c.av} = \dfrac{A_{core}}{A_{tot}}f_{c.core} + \dfrac{A_{cover}}{A_{tot}}f_{c.cover}$
(Megalooikonomou et. al. 2012) [5]	$\varepsilon_c = \dfrac{\Delta C}{C} = \dfrac{2\pi\{[R_{core}(1+\varepsilon_{r.core})+c(1+\varepsilon_{r.cover})]-(R_{core}+c)\}}{2\pi(R_{core}+c)}$ $= \dfrac{R_{core}(1+\varepsilon_{r.core})+c(1+\varepsilon_{r.cover})}{(R_{core}+c)}-1$

Table 1: Equations used in the iterative procedure

Figure 2 depicts a simple run of the material model under axial strain reversals with the same material properties as the specimen ST2NT of the experimental study with FRP- and steel- confined columns performed by Sheikh and Yau [6] . A moment-curvature analysis for the section (with layers/fibers) of the same specimen that provides also the shear force - shear angle diagram has been performed, where the constitutive model by Menegotto and Pinto [7] is used to model the longitudinal steel behavior. Figures 3, 4 and 5 depict the implications of the application of the constitutive relation presented in this paper, where in contrast to the assumption of a Timoshenko beam the shear deformation is not constant along the section. The shear deformation of the section derives as the mean value of the shear deformations of each material fiber/layer. The Bernoulli assumption is bypassed since the shear deformations are included and are uncoupled from the normal ones.

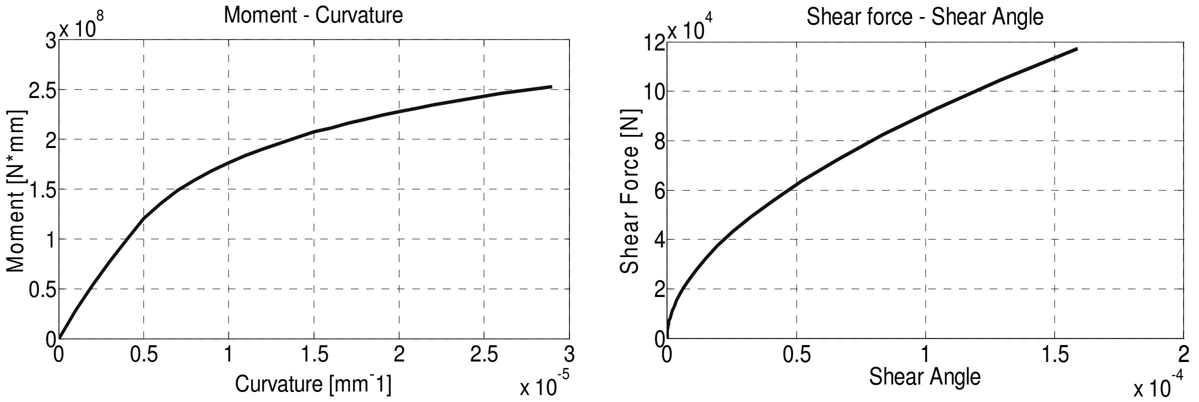

Figure 3: Prediction of section's response with properties similar to ST2NT by Sheikh and Yau [6]

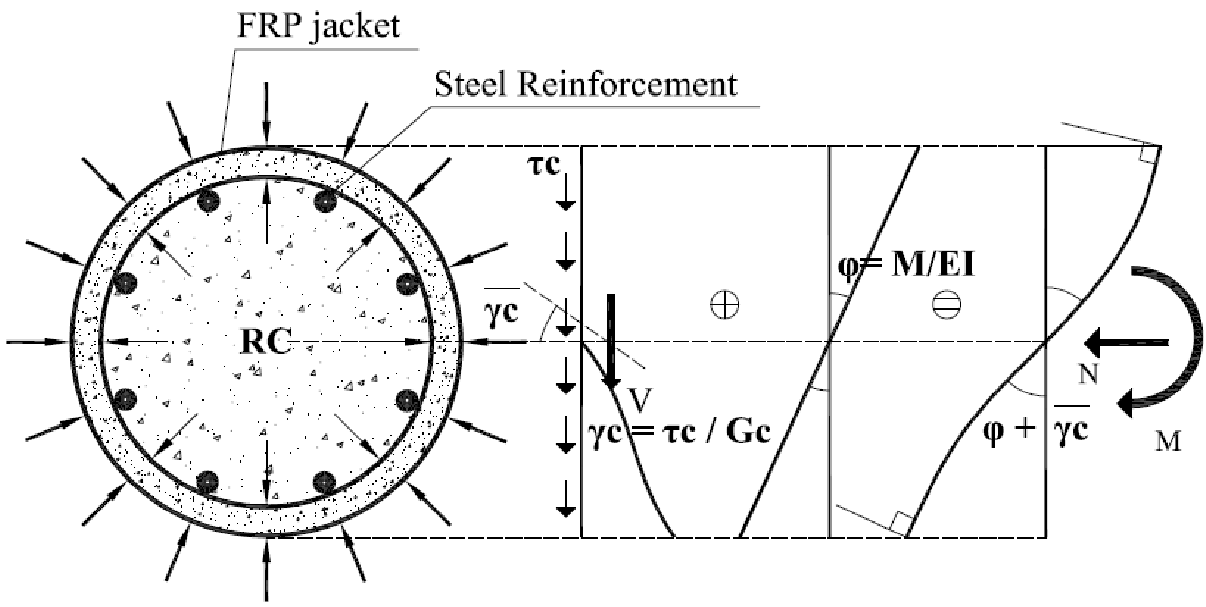

Figure 4: Circular concrete section confined by steel stirrups and FRP jacket under bending and shear.

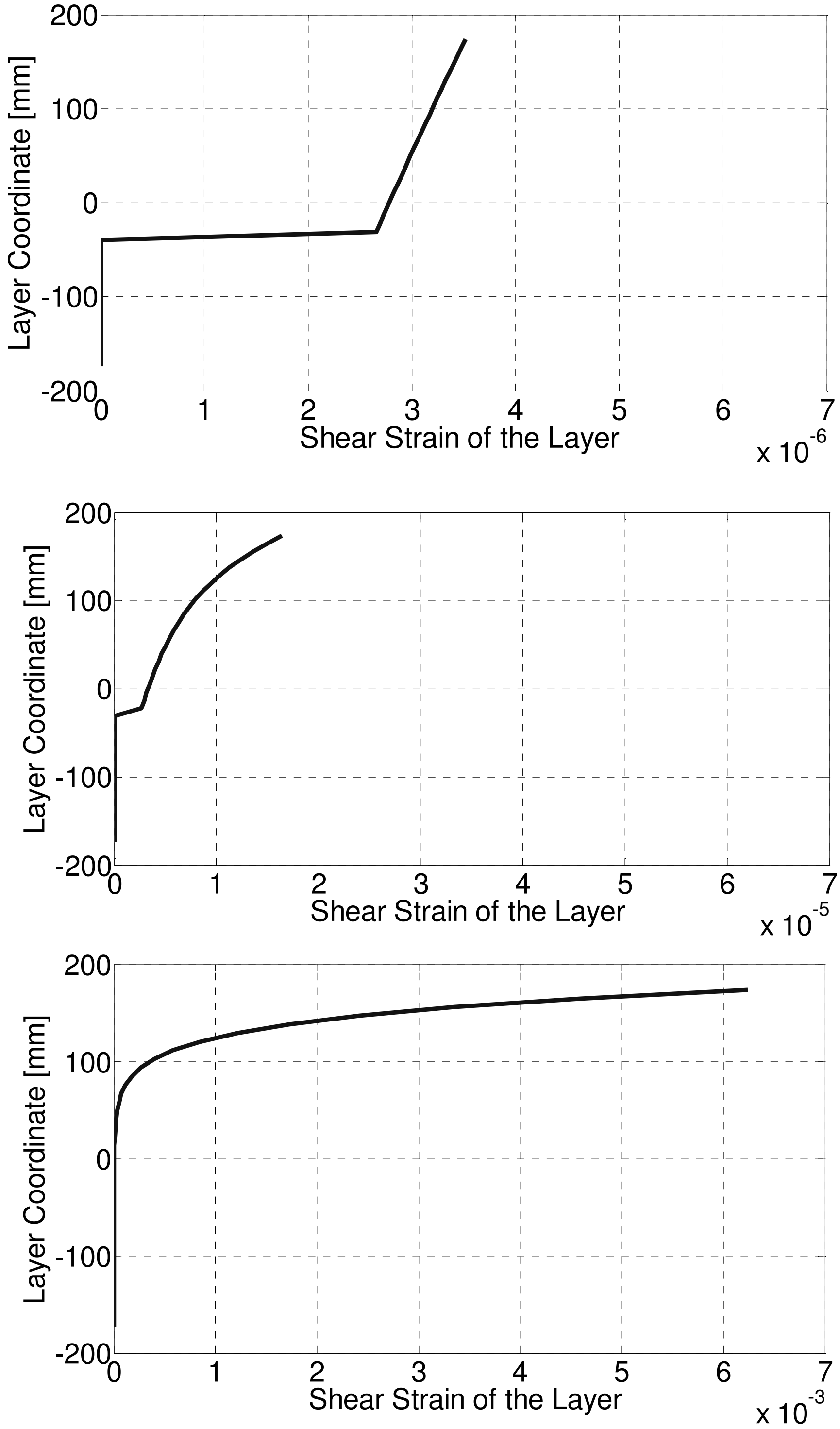

Figure 5: Shear strain profile over the section for the first curvature increment, flexural yielding and ultimate moment (from top to bottom).

3 COMPARISON OF PREDICTIONS AND EXPERIMENTAL RESULTS

The experimental study under consideration was performed by Sheikh and Yau [6] in which twelve 356 mm diameter and 1473 mm long columns were tested under constant axial load and reversed cyclic lateral load that simulated earthquake-induced forces. The test specimens were divided in three groups. The first group consisted of four columns that were conventionally reinforced with longitudinal and spiral steel reinforcement. The second group contained six reinforced concrete columns that were strengthened with carbon fiber-reinforced polymers (CFRP) or glass fiber-reinforced polymers (GFRP) before testing. The last group included two columns that were damaged to a certain extent, repaired with fiber-reinforced polymers (FRP) under axial load and then tested to failure. The correlation with the second group will be provided here. The columns contained six 25M longitudinal steel bars, and the spirals were made of U.S No 3. bars. The latter experimental program was conducted on FRP-retrofitted columns subjected to constant axial load and increasing cycles of lateral deformation in single-curvature setup. Four specimens of identical dimensions and steel reinforcements are used from this study. For each level of applied axial load (27% and 54% of the axial load carrying capacity, Po), two columns were retrofitted using two different types of FRP lamina, carbon (CFRP) and glass (GFRP), prior to testing.

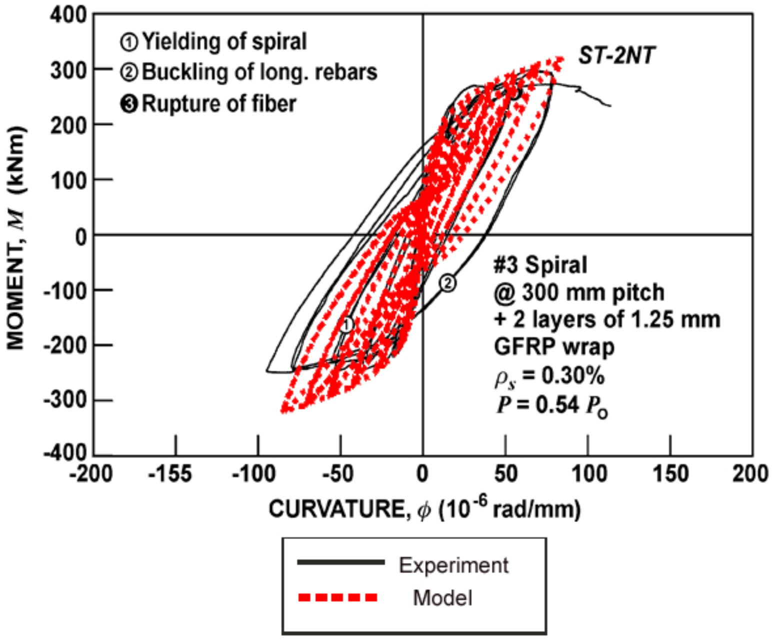

Figure 6: Correlation with experimental results ST-2NT – Sheikh and Yau [6]

The modeling of the RC columns has been performed using the “MatLab Finite Elements for Design Evaluation and Analysis of Structures” (FEDEAS Lab [8]). The experimental moment curvature responses within the plastic hinge regions are reported along with the numerical results in Fig. 6 – 9 (only flexure is considered in this case). The modeling of the cantilever columns has been applied using a unique fiber beam-column element [9] with displacement formulation for the entire column and then reporting the moment curvature response of the most critical section. It can be seen that the agreement is very close to the experimental one, with some deviation concentrated on the parts of reloading after reversal of the imposed displacement. This difference in response in terms of modeling can be explained based on the way the cracks on the concrete surface are described in the level of the material model. Because the crack is described as a two-event phenomenon (open or closed) (in reality, this is not the case due to imperfect crack closure), the contribution of concrete while the longitudinal steel reinforcement is in compression and the crack is closing gives this deviation in the response. The total moment-curvature response until the last step of numerical convergence is provided here.

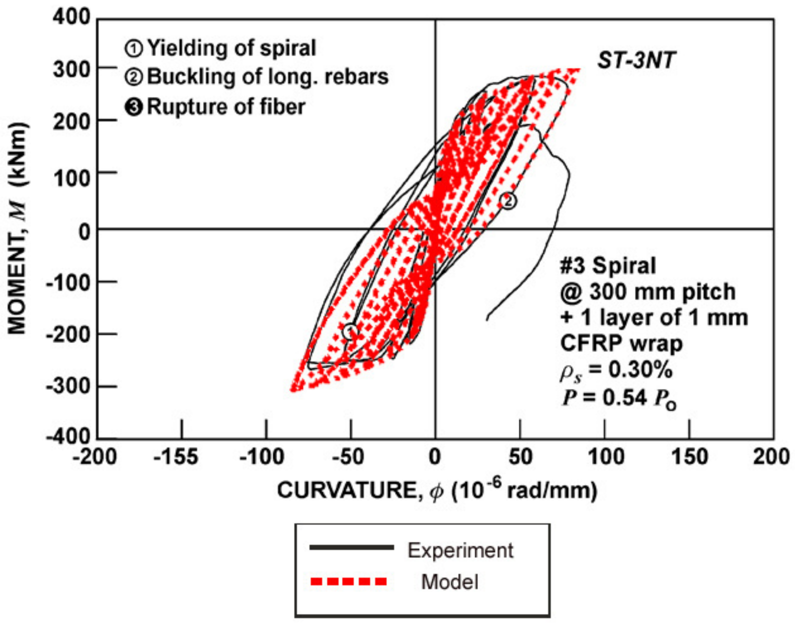

Figure 7: Correlation with experimental results ST-3NT – Sheikh and Yau [6].

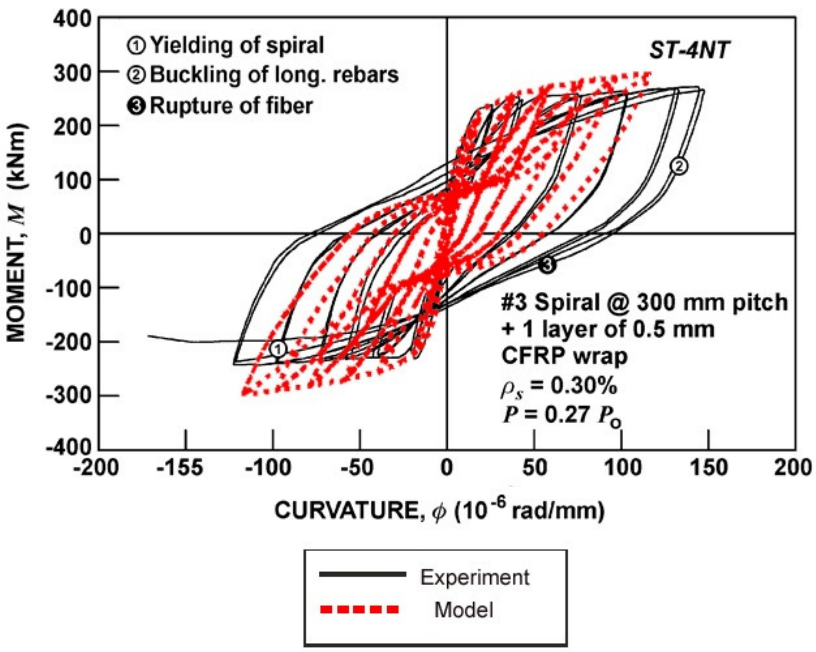

Figure 8: Correlation with experimental results ST-4NT – Sheikh and Yau [6].

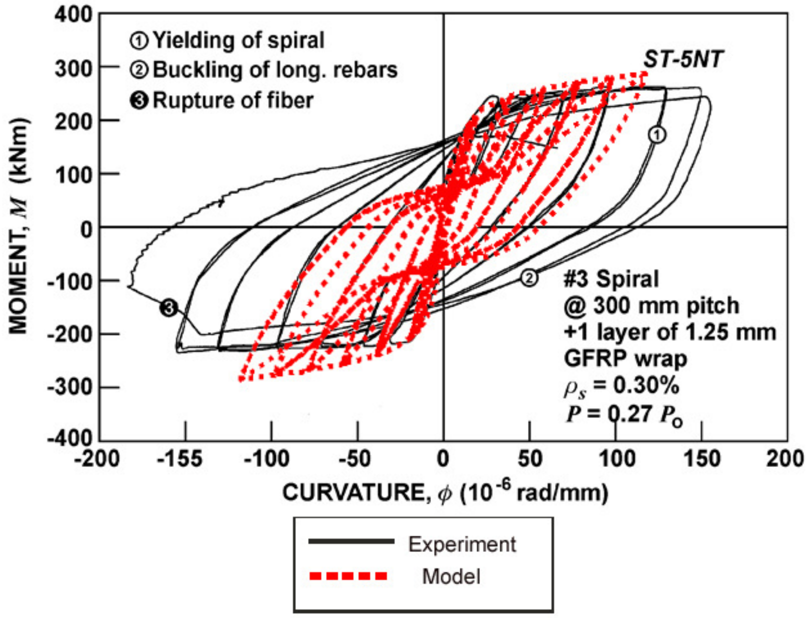

Figure 9: Correlation with experimental results ST-5NT – Sheikh and Yau [6].

4 CONCLUSIONS

Based on the results of this analytical investigation and the comparison of the predictions with the experimental study under consideration, the following conclusions are drawn:

- In case of RC column modeling with a fiber nonlinear beam- column element (displacement formulation) based on the presented constitutive law for concrete, apart from the immediate incorporation of shear deformations (uncoupled from the normal ones) in the material level (and in contrast to the standard fiber beam-column formulation), the averaged response of the two different regions -concrete core and concrete cover- in the cross-section allows the assignment of a unique stress-strain law for all the fibers/layers of the circular section.
- Moreover, in contrast to the assumption of a Timoshenko beam the shear deformation is not constant along the section. Along that line, also Timoshenko's assumption about shear deformation being a result of pure shear is adjusted in this case.
- Finally, another aspect that seems to be valid and important for further thought is that the response of the RC columns based on the model here presented is correct, although they are under cyclic excitation and contrary to the model assumptions, which are clearly static. Moreover, it uses the idea of the superposition of the effects of confinement that extends further of the linear assumptions.

ACKNOWLEDGEMENTS

This work has been carried out with the financial support of the Alexander S. Onassis Public Benefit Foundation.

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NOTATION:

C =*circumference of the circular section*

ΔC =*change in the circumference of the circular section*

ε_r =*radial strain*

ε_c =*circumferential strain*

ε_{con} =*concrete's axial strain*

ε_{co} =*concrete's axial strain at unconfined concrete's strength*

ε_{cc} =*concrete's axial strain at confined concrete's strength*

$\varepsilon_{c.core}$ =*circumferential strain of the core*

$\varepsilon_{r.core}$ =*radial strain of the core*

$\varepsilon_{r.cover}$ =*radial strain of the cover*

R_{core} =*radius of the concrete core*

c =*concrete cover*

f_{co} =*concrete strength*

f_{cc} =*confined concrete strength*

v_o =*initial Poisson's coefficient for concrete*

v =*Poisson's coefficient for concrete*

ρ_{sh} =*steel hoop's volumetric ratio*

ρ_j =*FRP jacket's volumetric ratio*

G_{core} =*shear modulus of concrete core*

G_c =*shear modulus of concrete*

τ_{core} =*shear stress of concrete core*

τ_c =*concrete shear stress*

γ_c =*concrete shear strain*

$\overline{\gamma_c}$ =*mean value of shear strains of the fibers/layers*

φ =*curvature of section*

M =*applied moment*

E = *modulus of elasticity*

I = *moment of inertia*

$f_{l.core}$ =*lateral confining pressure of the concrete core*

$f_{l.cover}$ =*lateral confining pressure of the concrete cover*

$f_{l.steel}$ =*lateral confining pressure of the steel reinforcement*

$f_{c.av}$ =*average axial concrete stress*

$f_{c.core}$ =*axial concrete core stress*

$f_{c.cover}$ =*axial concrete cover stress*

$f_{cc.core}$ =*axial confined concrete core strength*

$f_{cc.cover}$ =*axial confined concrete cover strength*

A_{core} =*area of concrete core*

A_{cover} =*area of concrete cover*

f_l =*lateral confining pressure of concrete*

f_c =*axial concrete stress*

E_{con} =*modulus of elasticity of concrete*

E_{sec} =*secant modulus of elasticity of concrete*

β =*a property of concrete evaluated as a function of the unconfined concrete strength*

D_b =*bar diameter*

E_s =*modulus of elasticity for steel*

A_{tot} =*total area of the circular section*

A_{sh} =*area of steel hoops (ties)*

ε_{sl} =*axial strain in the bar*

ε_{sh} =*steel hoop's strain*

ε_{shu} =*ultimate steel hoop's strain*

ε_{yh} =*steel hoop's strain for yielding*

k_{sl} =*Partial confinement coefficient for steel*

ξ_l =*coefficient taking into account longitudinal bar's confining effect*

ξ_{st} =*coefficient taking into account the confining effect of stirrups' spacing*

D_h =*hoop's diameter*

k_j =*partial wrapping coefficient*

A_b =*total area of longitudinal steel reinforcement*

A_g =*gross area of the section*

S_j =*jacket's clear spacing*

E_j =*jacket's modulus*

t_j =*thickness of the jacket*

P_o =*axial load capacity of column*

P =*axial load on column*